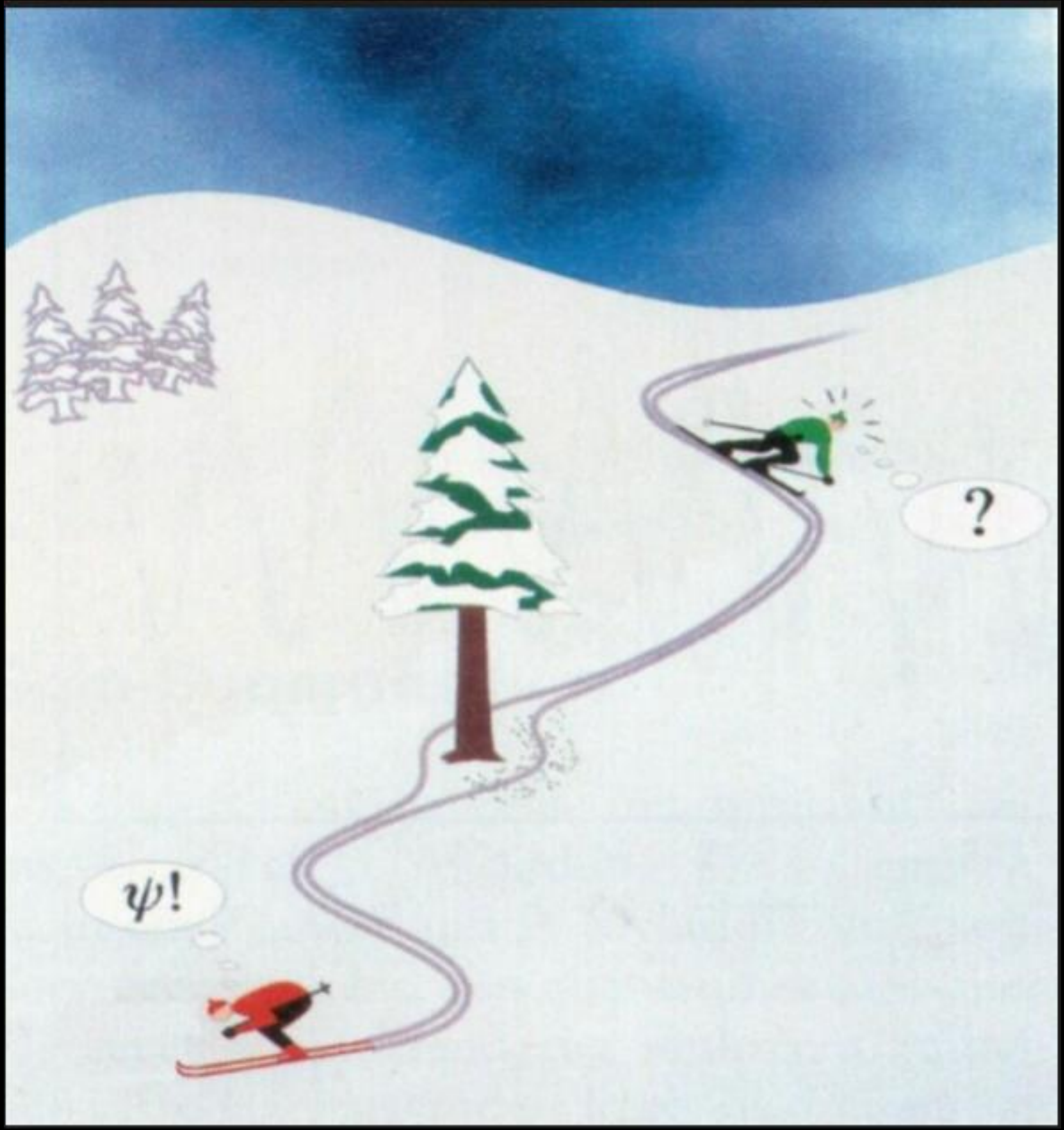
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ψ!

QUANTUM JUNCTION
GET IN BOTH LANES
Max 30²

MEASUREMENT = reading of the information through a special equipment to determine the status of the system

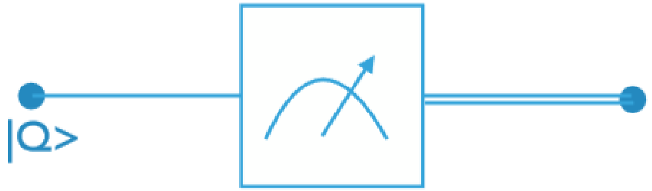
classic case: the measure is deterministic and does not alter the state of the bit

State of bit before the measurement	result of measurement	State of bit after the measurement
0	0	0
1	1	1

Probabilistic and **destructive**

Quantum measurement

State of bit before the measurement	result of measurement	State of bit after the measurement
$\|Q\rangle = a\|0\rangle + b\|1\rangle$	0 with probability $p_0 = \|a\|^2$ 1 with probability $p_1 = \|b\|^2$	$\|0\rangle$ $\|1\rangle$



Note: it explain the condition $|a|^2+|b|^2= 1$;
the sum of probability must be 1-

COMPOUND SYSTEM

2 bit: 4 alternatives

bit 1/bit2	0	1
0	00	01
1	10	11

2 qubit: 4 states corresponding to the classic one + superposition principle

2 qubit

$$|Q_1 Q_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

With a, b, c, d complex number and $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$

2 qubit logic gate

$$|Q_1Q_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

G ↓

$$|Q_1{}'Q_2{}'\rangle = a'|00\rangle + b'|01\rangle + c'|10\rangle + d'|11\rangle$$

With a', b', c', d' linear combination of a, b, c, d

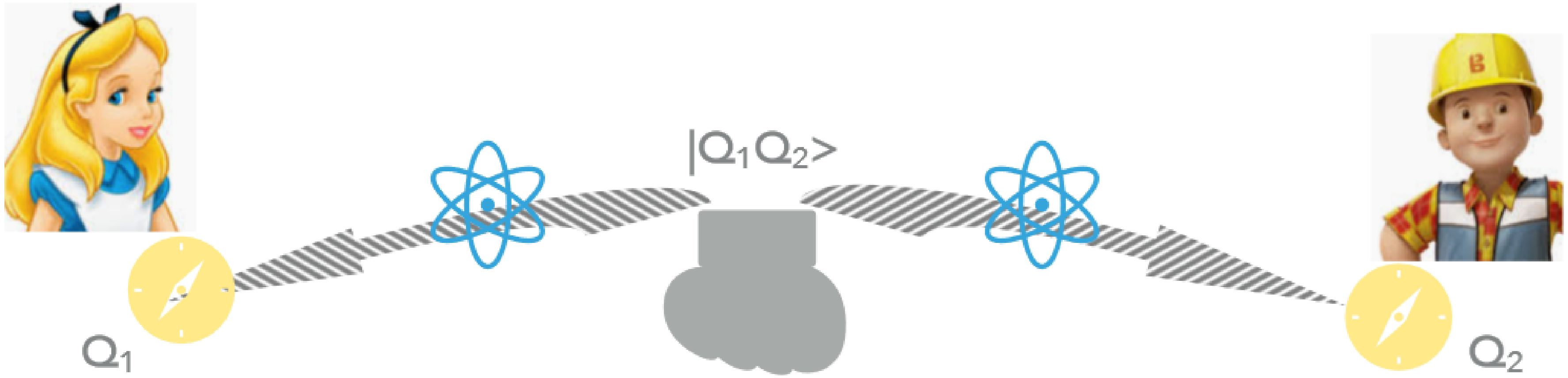
CNOT

Q_1Q_2	$Q'_1Q'_2$
00	00
01	01
10	11
11	10

Measurement

a measure on the qubit Q_1 allows us to determine if the first qubit is in $|0\rangle$ or $|1\rangle$ (similarly for Q_2), this measure is probabilistic and destructive

Examples. $|Q_1Q_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$



$a = b = {}^{1}/_{\sqrt{2}}\ , c = d = 0$ – produced state

State of bit before the measurement	State of bit after the measurement	result of measurement on Q_1	result of measurement on Q_2	$Q_1 = Q_2$
$\vert Q_1 Q_2\rangle = a\vert 00\rangle + b\vert 01\rangle$	$\vert 00\rangle$ with probability $\vert a\vert^2 = 1/2$ $\vert 01\rangle$ with probability $\vert b\vert^2 = 1/2$	0 0	0 1	YES NO

no correlation between Q_1 and Q_2

$a = d = {}^{1}/_{\sqrt{2}}\ , b = c = 0$ – entangled state

State of bit before the measurement	State of bit after the measurement	result of measurement on Q_1	result of measurement on Q_2	$Q_1 = Q_2$
$\vert Q_1 Q_2\rangle = a\vert 00\rangle + d\vert 11\rangle$	$\vert 00\rangle$ with probability $\vert a\vert^2 = 1/2$ $\vert 11\rangle$ with probability $\vert b\vert^2 = 1/2$	0 1	0 1	YES YES

Both the measurement on Q_1 and Q_2 is unpredictable

perfect correlation between Q_1 and Q_2

When it happens, the two qubit are entangled

Bell States
(maximally entangled)

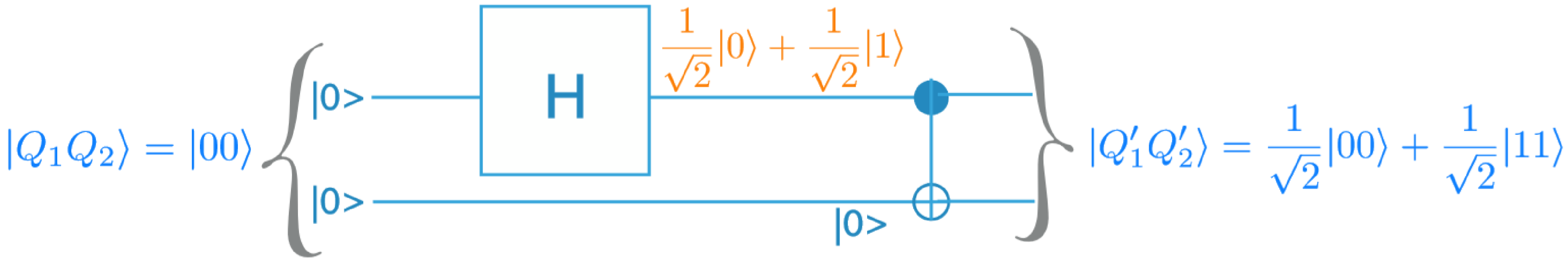
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

How to create entangled states



Exercise: what is the exit states if $|Q_1Q_2\rangle = |01\rangle, |10\rangle, |11\rangle$?